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Letters to the Editor

On the possibility of the existence of the de Broglie wavelength of second kind

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This note introduces the 4-dimensional version of the de Broglie wavelength which in turn implies that apart from the conventional de Broglie wavelength (which may be called the de Broglie wavelength of first kind) there must exist another de Broglie wavelength which has been termed the de Broglie wavelength of second kind. It has been discussed that the de Broglie wavelength of second kind offers a theoretical basis for the uncertainty relation $\Delta E\Delta t \sim \hbar$ which so far did not enjoy the status of the uncertainty relation $\Delta X\Delta P \sim \hbar$ the theoretical basis of which lies in the existence of the conventional de Broglie wavelength.

It may be recalled that in the literature on wave mechanics, the Klein-Gordon and the Dirac equations are very often referred to as the (relativistic) wave equations. There is, however, one formal difficulty in interpreting the Klein-Gordon and the Dirac equations as wave equations due to the non-availability of the wavelength appropriate for the equations concerned. No such difficulty is experienced in interpreting the Schrodinger equation as a wave equation due to the existence of the conventional de Broglie wavelength. Needless to mention that the wavelength associated with the Schrodinger equation is the (conventional) de Broglie wavelength To justify the wave equation status for the Klein-Gordon and the Dirac equations, we have introduced in this note the 4-dimensional version of the de Broglie wavelength which in turn implies that apart from the conventional de Broglie wavelength there must exist another de Broglie wavelength which has been termed the de Broglie wavelength of second kind. It may be recalled that the uncertainty relation $\Delta E\Delta t \sim \hbar$ does not enjoy the same theoretical status as its counterpart $\Delta X\Delta P \sim \hbar$ which was introduced by Heisenberg by exploiting the existence of the (conventional) de Broglie wavelength. The de Broglie wavelength of second kind, introduced in this note, offers the much desired theoretical basis for the uncertainty relation $\Delta E\Delta t \sim \hbar$ (which was introduced in analogy with the relation $\Delta X\Delta P \sim \hbar$).

To introduce the 4-dimensional version of the de Broglie wavelength we shall employ the arguments based on the invariance principle. The necessity of the invariance properties of physical quantities is well known (Sakurai 1964) and we have discussed this point elsewhere (Mukhopadhyay 1975). We examine first the invariance property of the de Broglie wavelength λ_D defined by

$$\lambda_D = h/P. \qquad \dots \quad (1)$$

The invariance property of λ_D becomes more transparent if we rewrite eqn (1) in the following form

$$\lambda_D = h/P = h/(\mathbf{P}.\mathbf{P})^{\frac{1}{2}} = h/(P_\mu P^\mu)^{\frac{1}{2}}, \ \mu = 1, 2, 3. \qquad \dots \quad (1a)$$

Since $P^2 = P_\mu P^\mu$ is invariant under 3-dimensional rotations, therefore, P (the magnitude of 3-momentum) is so. This is in turn suggests that the de Broglie wavelength λ_D is a scalar and as such an invariant in 3-dimensional space. The invariance property of the de Broglie wavelength clearly suggests that it is a well defined physical quantity in 3-dimensional space in which the Schrodinger equation is valid. In this connection we may note that the Schrodinger equation is covariant (and as such form invariant) under 3-dimensional rotations. This fact suggests that as the space in which the Klein-Gordon and the Dirac equations are valid is 4-dimensional, therefore, the wavelength underlying the equations concerned has to satisfy the requirement of Lorentz invariance. It may be noted that eqn. (1a) has been expressed in the form which can be easily generalized to 4-dimensional space by allowing μ to run from 0 to 3. It is easy to write down the 4-dimensional version of eqn (1a) which reads

$$\lambda = h/(p_\mu p^\mu)^{\frac{1}{2}}, \quad \mu = 0, 1, 2, 3 \qquad (2)$$

where λ is the wavelength appropriate for the waves propagating in 4-dimensional space. Needless to mention that the norm of 4-momentum p_μ is invariant under 4-dimensional rotations as $p_\mu p^\mu$ is so. Obviously, then, λ is a Lorentz invariant quantity and as such it is a scalar in 4-dimensional space. Using the well known relation (Bjorken & Drell 1964) $p_\mu p^\mu = m^2c^2$, eqn. (2) can be given the following form

$$\lambda = h/(mc) \qquad \dots \quad (2a)$$

where m is the Lorentz invariant mass (i.e. the rest mass). It may be noted that as eqn. (2a) has been obtained by generalizing eqn. (1a) to 4-dimensional space, therefore, λ occurring in eqn. (2a) is the 4-dimensional analogue of λ_D defined by eqn. (1a). It may be emphasized here that the presence of the mass m in eqn. (2a) implies that λ is the wavelength of the waves which represent a massive particle executing 4-dimensional motion. Obviously, then, λ is the wavelength appropriate for the Klein-Gordon and the Dirac equations.

To verify that λ, defined by eqn. (2a), is the wavelength associated with the Klein-Gordon and the Dirac equations we proceed as follows. For our purpose we start from the time-independent Schrodinger equation

$$[\nabla^2 + 2mE/\hbar^2]\psi = 0 \qquad \dots \quad (3)$$

which can be recast in the following form with the help of the relation $\lambda_D = h/P = h/(2mE)^{\frac{1}{2}}$

$$[\nabla^2 + (4\pi^2)/\lambda_D^2]\psi = 0. \qquad \ldots \quad (3a)$$

The time-dependent Schrodinger equation can also be reduced to the form, given by eqn. (3a), using the transcription $i\hbar(\partial/\partial t) \rightarrow E$ which transforms the time-dependent Schrodinger equation into its time-independent form, given by eqn (3), which ultimately can be recast in the form of eqn. (3a). We have already noted that λ, defined by eqn. (3a), is the analogue of the de Broglie wavelength λ_D in 4-dimensional space. Denoting by ϕ the analogue of the Schrodinger-ψ in 4-dimensional space and using the transcriptions $\nabla^2 \rightarrow \Box$, $\lambda_D \rightarrow \lambda$ and $\psi \rightarrow \phi$, the 4-dimensional version of eqn. (3a) takes the following form

$$[\Box + (4\pi^2)/\lambda^2]\phi = 0. \qquad \ldots \quad (4)$$

Eqn. (4) can be rewritten with the help of eqn. (2a) as shown below.

$$[\Box + m^2c^2/\hbar^2]\phi = 0 \qquad \ldots \quad (4a)$$

which is the free particle Klein-Gordon equation The way the Klein-Gordon equation has been obtained starting from the Schrodinger equation clearly reveals that λ, defined by eqn. (2a), is the wavelength associated with the former equation. Either from Lorentz invariance arguments or from physical considerations, it follows that the wavelength associated with the Klein-Gordon equation is also the wavelength appropriate for the Dirac equation This is so because, as it is well known, each and every component of the Dirac-ψ must necessarily satisfy the Klein-Gordon equation for the requirement of energy-momentum conservation. There is hardly any need of emphasizing that just as the de Broglie wavelength imparts the wave equation status on the Schrodinger equation similarly its 4-dimensional version i.e. the wavelength λ, defined by eqn. (2a), implies the same for the Klein-Gordon and the Dirac equations We repeat to emphasize that the wave equation status is clearly justified for the Klein-Gordon equation, namely, eqn. (4a) which was obtained, as shown above, starting from the Schordinger equation which is a true wave equation. This fact automatically implies the wave equation status also for the Dirac equation for the reasons mentioned above

In the above we have introduce the 4-dimensional analogue of the de Broglie wavelength. Now we want to demonstrate that the 4-dimensional version of the de Broglie wavelength demands the existence of the de Broglie wavelength of second kind apart from the conventional de Broglie wavelength (which may be called the de Broglie wavelength of first kind). For our purpose we proceed as follows. It is well known that a Lorentz covariant description of a particle in its 4-momentum space requires 4-momentum $p_r = (\boldsymbol{P}, p_0)$ where $\boldsymbol{P}$ and p_0

are the space-like and the time-like components respectively of p_μ. In the pseudo-Euclidean space (Schweber 1961, Bjorken & Drell 1964) with the metric $g_{\mu\nu}$ such that $g_{00} = -g_{11} = -g_{22} = -g_{33} = +1$ and $g_{\mu\nu} = 0$ when $\mu \neq \nu$, the time-like component p_0 can be taken to be real and defined by $p_0 = E/c$. It is interesting to note that the conventional de Broglie wavelength $\lambda_D = h/P$ is related to the magnitude of the space-like component $\boldsymbol{P}$ of 4-momentum p_μ. Also we have shown that the norm of 4-momentum p_μ is related to the 4-dimensional version of the de Broglie wavelength Obviously, the following equations

$$\lambda = h/(p_\mu p^\mu)^{\frac{1}{2}} = h/(\text{norm of } p_\mu)$$

$$\lambda_D = h/P = h/(\text{magnitude of } \boldsymbol{P})$$

clearly suggest that there must be a wavelength λ_S associated with the magnitude of p_0, the time-like component of p_μ The wavelength λ_S is defined by

$$\lambda_S = h/p_0 = h/(E/c). \tag{5}$$

Since

$$E = (P^2c^2 + m^2c^4)^{\frac{1}{2}},$$

therefore, eqn. (5) can be rewritten as

$$\lambda_S = h/(P^2 + m^2c^2)^{\frac{1}{2}}. \tag{5a}$$

The wavelength λ_S may be called the de Broglie wavelength of second kind. For a free massive particle $(p_\mu p^\mu)^{\frac{1}{2}}$, P and p_0 are separately conserved and as such λ, λ_D and λ_S are physically meaningful quantities. Obviously, the quantities $\lambda_x = h/P_x$, $\lambda_y = h/P_y$ $\lambda_z = h/P_z$ are not physical quantities as P_x, P_y, P_z are not constants of motion.

The intimate connection between Heisenberg's uncertainty relation $\Delta X \Delta P \sim \hbar$ and de Broglie's wavelength is well known (Powell & Crasemann 1964). In this connection it may be noted that the relation $\Delta E \Delta t \sim \hbar$ was introduced in analogy with the relation $\Delta X \Delta P \sim \hbar$ It may be noted, however, that the relation $\Delta E \Delta t \sim \hbar$ does not enjoy the respectable theoretical status of Heisenberg's relation $\Delta X \Delta P \sim \hbar$ which has, as its basis, the conventional de Broglie wavelength λ_D It is gratifying to note that λ_S, the de Broglie wavelength of second kind, introduced in this note offers a theoretical basis for the relation $\Delta E \Delta t \sim \hbar$. This statement becomes more transparent if we rewrite the relation $\Delta E \Delta t \sim \hbar$ as $\Delta(E/c)\Delta(ct) \sim \hbar$ which in turn can be recast as $\Delta p_0 \Delta x_0 \sim \hbar$ where $x_0 = ct$ (Schweber 1961) is the time component of $x_\mu = (\boldsymbol{X}, x_0 = ct)$. Eqn. (5) relates the wavelength λ_S with the time-like component p_0 of 4-momentum p_μ In a future communication we shall discuss further applications of the de Broglie wavelength of second kind.

The author humbly dedicates this work to the memory of his parents—the late Sm Sarojbasini Mukhopadhyay and the late Sri Harimohon Mukhopadhyay.

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Piezoelectric effect in sodium benzoylacetonate compound

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The aim of this work is to study the piezoelectric effect of sodium benzoylacetonate in order to throw some light on the relation between the piezoelectric effect in this compound and that in sodium acetylacetonate materials (Tawfik 1975).

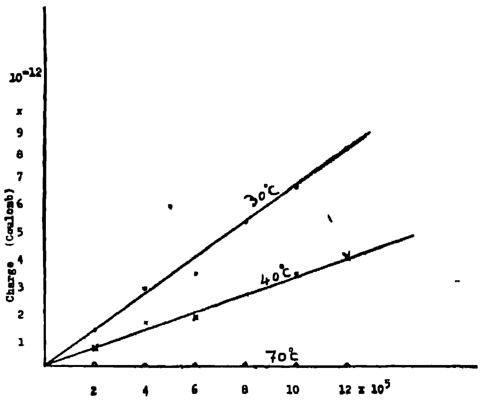

Fig. 1